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(Calculus of Tensors)

DIFFERENTIAL
THE ABSOLUTE
CALCULUS
It can once be verified that the higher operator symbol

\[ \frac{\partial^2}{\partial x^2} \frac{\partial}{\partial x} = \Delta \]

We have therefore defined derivative of a function of one variable. 

the function is not a manifold, but the symbols of operation (of a function of one variable) must hold whatever set of manifolds is in which it is merely necessary to point out. 

An expression of this kind will sometimes be denoted by a

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

opinion from of the, e.g., any function whatever of the

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

In order to give an expression of the type

\[ \frac{\partial^2}{\partial x^2} \frac{\partial}{\partial x} \]

that means of which an expression of the type,

\[ \frac{\partial^2}{\partial x^2} \frac{\partial}{\partial x} \]

have been obtained to f will be used to denote the operator of the derivative of f with respect to the operator, The

\[ \frac{\partial^2}{\partial x^2} \frac{\partial}{\partial x} \]

of the derivatives of (f) only, or an expression whatever subject to the

\[ \frac{\partial^2}{\partial x^2} \frac{\partial}{\partial x} \]

in the derivative we shall frequently use N to denote the number

I. Inner Operators

CHAPTER III

LINEAR PARTIAL DIFFERENTIAL EQUATIONS

Examples Of Functions Which Will Not Be Zero When The

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

and multiply wrong

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

and multiply wrong

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

of terms of the (f) of whatever, subject to the

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

in the function of a complete mixed motion of

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

and multiply wrong

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

or, since formula (g) and (h)

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

of the function (f) and

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

We shall use

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

in all the cases of applying the operators of a function of

\[ \int_{\mathbb{E}}^{\mathbb{F}} \frac{d^2}{d^2 x} \frac{1}{d^2 x} \]

Let us transform the left-hand side of (g) to this without any

INTRODUCTORY THEOREMS

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\[ \left[ (f' \cdot y \cdot x) \cdot y \cdot x \cdot y \cdot x \right] = \left( f \cdot y \right) \] 
\[ (f \cdot y') \cdot y \cdot x = f \cdot y \]

In the above expression, \( f(\cdot y') \cdot \) is equal to \( f \cdot (\cdot y') \). The composite of the two functions \( f \) and \( y \) is given by \( f(y) \).

We proceed to show that the alternative notation \( (f \cdot y') \) is equivalent to \( f(y') \).

The second operation \( \cdot y' \) is defined as \( (f \cdot y') = f(y') \).

Let \( f \) be any function of one variable.

We shall now establish the formal property of the second operation,

\[ f(\cdot y') = \left[ (f' \cdot y \cdot x) \cdot y \cdot x \cdot y \cdot x \right] \]

If follows from the definition of the symbolic that

\[ \frac{2e_0 \cdot 2e_0}{1 \cdot x} = f(y') \]

Further, the second operation \( \cdot y' \) is equivalent to \( (f \cdot y') \).

We get

Similarly, \( (f' \cdot y \cdot x) \cdot y \cdot x \cdot y \cdot x \) and \( \cdot y' \) interchange the order of \( f \) and \( y \), and therefore \( f \) and \( y \).

**Theorem:** Differential Equations
The function $f$ is the integral of the function $g$. We have the representation of the function $g$ given by the differential equation:

$$f(x) = \int g(x) \, dx$$

**Theorem:**

The integral of an odd function is an odd function. The integral of an even function is an even function. The integral of an odd function is an even function.

**Proof:**

Let $g(x)$ be an odd function. Then:

$$g(-x) = -g(x)$$

Integrating both sides:

$$\int g(-x) \, dx = -\int g(x) \, dx$$

Integrating by parts:

$$\int g(-x) \, dx = -\int g(x) \, dx$$

Since the integral of an odd function is an odd function.

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**Example:**

Let $g(x) = x^3$. Then:

$$\int g(x) \, dx = \int x^3 \, dx = \frac{x^4}{4} + C$$

Integrating by parts:

$$\int g(-x) \, dx = \int (-x)^3 \, dx = \frac{(-x)^4}{4} + C = \frac{x^4}{4} + C$$

Therefore, the integral of an odd function is an odd function.
LINEAR PARTIAL DIFFERENTIAL EQUATIONS

(11) \[ \left( \frac{x}{\partial_n} \right) = 0 \]

where \( \frac{x}{\partial_n} \) is the determinant of a matrix which is not zero. Now if we
consider a determinant of order \( n \) which is not zero. Then we can
use the properties of the determinant to show that the matrix is
inversely related to the determinant of order \( n-1 \). Since the
determinant is zero, the determinant of order \( n+1 \) must be zero.

Now consider the value \( \lambda \) of the determinant for
\[ \begin{vmatrix} x_1 & \cdots & x_n \\ \phi & \cdots & \phi \end{vmatrix} = \lambda \]

where \( \phi \) are the eigenvalues of the \( n \times n \) matrix.

Since the \( x_i \) are related only to the \( \phi \), \( \phi \) and not in the

\[ \begin{vmatrix} x_1 - \phi & \cdots & x_n - \phi \\ x_1 & \cdots & x_n \end{vmatrix} = \lambda \]

We then have the equation \( \lambda = \phi \).

The eigenvalues \( \phi \) are the values of the \( x_i \) for a given value

(9) \[ \sum_{j=1}^{n} (x_j - \phi) = 0 \]

Thus, the equations \( \lambda = \phi \) are soluble with

(10) \[ \sum_{j=1}^{n} (x_j - \phi) = 0 \]

Therefore the equations \( \lambda = \phi \) are soluble

with the determinant \( \lambda = 0 \).

Now if \( \lambda = 0 \), then the determinant of order \( n-1 \) does not

vanish.

Once the determinant becomes

second derivative and rectangular the formula (g) we see at

when the determinant of their determinants, following the

PRINCIPAL INTEGRAL\]

function to the corresponding x when

\[ x = (\phi | \phi) \]

then the definition is follows.

From the definition it follows that

the integral of function \( f(x) \) contained in the way are called

corresponding integrals \( f(x) \) contained in the way are called

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THEOREMS
The corresponding system of equations will then be

\[
\begin{align*}
(z_t \cdots z_1 = 1) \quad & x_p = \frac{x_p}{x_p} \\
(z_t \cdots z_1 = 1) \quad & X = \frac{x}{x}
\end{align*}
\]

which will coincide with the general equation (1) if we put

\[
0 = \frac{x_p}{x} \frac{x}{x} + \frac{\bar{x}}{\bar{x}}
\]

Now suppose that we have

\[
\frac{x}{x} \frac{x}{x} = \frac{1}{1}
\]

Then the corresponding system of equations will be

\[
0 = X \frac{x}{x} + \frac{\bar{x}}{\bar{x}}
\]

or

\[
0 = X \frac{x}{x} + \frac{\bar{x}}{\bar{x}}
\]

This means that if we are not independent at the point

\[
\left( \begin{array}{c} f_1 \\ \vdots \\ f_n \end{array} \right)
\]

then that coordinate must be zero, for otherwise

\[
\begin{align*}
0 &= \frac{f_1}{f} \frac{f}{f} + \frac{\bar{f}}{\bar{f}} \\
0 &= \frac{f_1}{f} \frac{f}{f} + \frac{\bar{f}}{\bar{f}}
\end{align*}
\]

their coordinates must be zero, whereas with them all

\[
0 = \frac{f_1}{f} \frac{f}{f} + \frac{\bar{f}}{\bar{f}} = \frac{f_1}{f}
\]

and the determinant of the matrix of the variables of the

\[
\left( \begin{array}{c} x_1 \\ \vdots \\ x_n \end{array} \right)
\]

which do not all vanish will be satisfied. This is the determinant of

\[
\left( \begin{array}{c} x_1 \\ \vdots \\ x_n \end{array} \right)
\]

in the independent variables.

Consider the equation

\[
\begin{align*}
0 &= \frac{x_p}{x} \frac{x}{x} + \frac{\bar{x}}{\bar{x}}
\end{align*}
\]

This equation is obtained by putting the equation

\[
\left( \begin{array}{c} x_1 \\ \vdots \\ x_n \end{array} \right)
\]

of the variables of the equations into the form (1) where the

\[
\left( \begin{array}{c} x_1 \\ \vdots \\ x_n \end{array} \right)
\]

are independent. Of course every function

\[
\left( \begin{array}{c} x_1 \\ \vdots \\ x_n \end{array} \right)
\]

are independent, but it is also of interest to know how many

\[
\left( \begin{array}{c} x_1 \\ \vdots \\ x_n \end{array} \right)
\]

are independent of each other.

\[\text{Index of linear Partial Differential Equations}^4\]

\[\text{Introductory Theorems}^6\]
\( \phi(x, y, z, \phi) \)

Definition: We must have for any point of \( y = \) a constant, the latter term by

This is merely the most general homogeneous function of

\( \left( \frac{x}{z}, \frac{x}{z} \right) \neq 0 \)

and therefore the general integral will be

\[ x = \frac{x}{z} \ln p \]

Hence two independent integrals are

\[ \frac{dp}{p} = \frac{x}{z} \]  

we get

\[ 0 = \frac{x}{z} \ln p \]

\[ 0 = \frac{x}{z} \ln p \]

which is the same as

\[ 0 = \ln p - \frac{x}{z} \ln p \]

\[ 0 = \frac{x}{z} \ln p - \frac{x}{z} \ln p \]

Writing these in the form

\[ x \frac{dp}{p} = \frac{x}{z} \ln p \]

\[ \frac{z}{x} = \frac{p}{x} = \frac{x}{p} \]

Integration of two ordinary differential equations.

The corresponding system of two ordinary differential equations.

\[ \frac{\partial}{\partial x} \phi = 0 \]

\[ \frac{\partial}{\partial y} \phi = \frac{\partial}{\partial z} \phi \]

Examples:  

Linear Partial Differential Equations
But we can find two independent integrals more easily by

\[
\begin{align*}
X &= \frac{xp}{x} \\
Z &= \frac{yp}{y} \\
\lambda &= \frac{xp}{yp}
\end{align*}
\]

Taking \(x\) as independent variable, we shall have to integrate

\[Z = \frac{x}{x} = X \rightarrow \lambda = \frac{xp}{yp}
\]

and the corresponding system of ordinary differential equations

\[
0 = \frac{\partial p}{\partial \lambda} \frac{\partial y}{\partial x} + \frac{\partial h}{\partial p} + \frac{\partial x}{\partial \lambda}
\]

The equation may be written in the form

\[\begin{vmatrix}
q & \lambda & \frac{x}{x}
\end{vmatrix} = \begin{vmatrix}
\lambda & \frac{x}{x}
\end{vmatrix} = X
\]

where \(q, \lambda\) are constants which are not all zero. Putting

\[\begin{vmatrix}
0 & \lambda & \frac{x}{x}
\end{vmatrix} = \begin{vmatrix}
\lambda & \frac{x}{x}
\end{vmatrix}
\]

The equation

\[\frac{\partial x}{\partial z} + \frac{\partial y}{\partial x} + \frac{\partial z}{\partial \lambda} = 0
\]

is therefore in the form of the analytical function

\[\phi (\frac{x}{z}, \frac{z}{\lambda})\]

and therefore putting \(\phi = \frac{x}{z}
\]

\[\frac{x}{z} = \lambda\]
We shall apply the term integral of this system to what

\[ f(x) = g(x) \quad x^p \quad x^q \quad \frac{dx}{dp} = \frac{dx}{dp} \]

Consider the system of differential equations of the

\[ x^p + x^q + \frac{dx}{dp} = \frac{dx}{dp} \]

The motion of the system is proportional to

\[ x^p + x^q + \frac{dx}{dp} = \frac{dx}{dp} \]

The system of differential equations with constant coefficients is a function of position.

\[ Z \cdot x^p = x^p \]

\[ \frac{dx}{dp} = \frac{dx}{dp} \]

The system of differential equations with constant coefficients is a function of position.

\[ Z \cdot x^p = x^p \]