The purposes of this analysis were to predict the feasible movements during which balance can be maintained, based on environmental (contact force), anatomical (foot geometry), and physiological (muscle strength) constraints, and to identify the role of each constraint in limiting movement. An inverted pendulum model with a foot segment was used with an optimization algorithm to determine the set of feasible center of mass (CM) velocity-position combinations for movement termination. The upper boundary of the resulting feasible region ran from a velocity of 1.1 s\(^{-1}\) (normalized to body height) at 2.4 foot lengths behind the heel, to 0.45 s\(^{-1}\) over the heel, to zero over the toe, and the lower boundary from a velocity of 0.9 s\(^{-1}\) at 2.7 foot lengths behind the heel, to zero over the heel. Forward falls would be initiated if states exceeded the upper boundary, and backward falls would be initiated if the states fell below the lower boundary. Under normal conditions, the constraint on the size of the base of support (BOS) determined the upper and lower boundaries of the feasible region. However, friction and strength did limit the feasible region when friction levels were less than 0.82, when dorsiflexion was reduced more than 51%, or when plantar flexion strength was reduced more than 35%. These findings expand the long-held concept that balance is based on CM position limits (i.e., the horizontal CM position has to be confined within the BOS to guarantee stable standing) to a concept based on CM velocity-position limits.
The objective of the optimization was to identify the boundaries of all possible velocity-position combinations which may satisfy the task conditions: The mass must arrive at a position over the BOS as the velocity vanishes, while the feet remain stationary. Thus, at any given CM position, two boundary velocities were defined, beyond which the task could not be accomplished without foot motion:

1. The largest tolerable velocities that still can be reduced to zero before the CM exceeds the anterior limit of the BOS.
2. The smallest necessary velocities that still allow the CM to reach the posterior limit of the BOS.

Because the foot was not rigidly attached to the floor, the model's ability to transmit forces from the floor to the BOS was limited by three constraints:

1. Gravity constraint: The net vertical ground reaction force, \( F_{gy} \), must be positive:
   \[ F_{gy} \geq 0 \]  
2. Friction constraint: The horizontal ground reaction force, \( F_{gx} \), must not exceed the slip threshold dictated by the coefficient of friction, \( m \):
   \[ |F_{gx}| < \mu F_{gy} \]  
3. Center of pressure (COP) constraint: The COP must reside within the length of the BOS, if:
   \[ 0 < \text{COP} < l \]  

Each of these inequality equations was then algebraically related to the dynamics of the pendulum (Appendix), and expressed in terms of the angular position, \( \Theta \), angular velocity, \( \dot{\Theta} \), and resultant ankle torques, \( \tau \), resulting in boundaries in the state-torque space:

\[
\begin{align*}
\tau_{\text{COP}, \text{post}} &> f_{\Theta}(\Theta, \dot{\Theta}) \\
\tau_{\text{COP}, \text{anterior}} &> f_{\Theta}(\Theta, \dot{\Theta}) \\
\tau_{\text{COP}, \text{posterior}} &< f_{\Theta}(\Theta, \dot{\Theta}) \\
\tau_{\text{COP}, \text{anterior}} &< f_{\Theta}(\Theta, \dot{\Theta})
\end{align*}
\]  

In addition to these constraints, the resultant ankle torques, \( \tau \), were physiologically constrained to a range of values under maximum muscle strength:

\[ (\tau)_{\text{maximum, plantarflexion}} \leq \tau \leq (\tau)_{\text{maximum, dorsiflexion}} \]  

The maximum values for the \( \tau \) were assumed to be dependent on joint position, and were taken from a musculoskeletal model (see Figure 6, page 761 in Delp et al., 1990). This Hill-based model considers the force-length properties of muscles crossing the ankle, the lines of action of all musculotendonous elements, and sums their effect to estimate the maximum dorsiflexion and plantar flexion torques.
A switching state feedback controller (Figure 2, solid lines) considered each constraint (Equations 4 and 5), and determined the maximum and minimum feasible torque necessary to keep the feet stationary (Figure 2, solid lines). Using this controller, two optimal CM velocity-position trajectories were obtained iteratively by adjusting the initial states (Figure 2, broken lines). These optimal trajectories either maximized or minimized the initial velocity that still allowed the task to be completed successfully (i.e., the CM arrived precisely above the anterior or posterior limit of the BOS as the velocity vanished, while the feet remained stationary). The trajectory with the “fastest” initial velocity at the most posterior initial position was used to define the fall-forward boundary, and the trajectory with the “slowest” initial velocity at the most posterior initial position was used to define the fall-backward boundary. A fifth order Runge-Kutta method with a fourth order step-size control (MATLAB, Math Works, Inc., Natick, Mass, U.S.A.) was used to integrate the equation of motion (Equation A1, Appendix).

Additional analyses were conducted to calculate the threshold values where key parameters started to affect the feasible region, and to identify the effects of extreme conditions. Consequently, friction was reduced iteratively to an extreme value of $\mu = 0.05$ to simulate slippery conditions (oil on metal, cf. Chaffin et al., 1992; Lloyd and Stevenson 1992). Similarly, strength was reduced 59% to simulate the weakness in an elderly group (elderly mean minus one standard deviation in Gerdle and Fugl-Meyer, 1985), and because the COP range observed in elderly subjects during functional activities can be much smaller than the full foot length, the range of the BOS was reduced 55% (an extreme case in Lee and Deming, 1988).

**Figure 1. Pendulum and foot model with free body diagrams for the segments.**

**Figure 2. Optimization scheme.** A switching state feedback controller (using the bounding surfaces to determine the maximum and minimum feasible torque for a given state [solid lines]), obtained two optimal center of mass velocity-position trajectories iteratively by adjusting the initial states (broken lines).

**Results**

The upper boundary of the resulting feasible region ran from a velocity of $1.1 \text{ s}^{-1}$ (normalized to body height) at 2.4 foot lengths behind the heel, to 0.45 s$^{-1}$ over the heel, to zero over the toe, and the lower boundary ran from a velocity of 0.9 s$^{-1}$ at 2.7 foot lengths behind the heel, to zero over the heel (gray area, Figure 3). Forward falls would be initiated if states exceeded the upper boundary, and backward falls would be initiated if the states fell below the lower boundary.

**Figure 3. Feasible horizontal center of mass velocity-position region (shaded diagonal band) for terminating anterior movement of a simple pendulum connected to a stationary base of support.** Forward falls would be initiated if states exceeded the upper boundary, while backward falls would be initiated if states dropped below the lower boundary. The initiation of a fall was also defined in this analysis as any dynamic condition that causes the feet to move. Velocities and positions were normalized by body height and foot length, respectively (toe is zero and heel is 1). The bold arrow indicates the direction a trajectory would travel in terminating movement.

Under normal conditions, the COP_{toe} constraint determined the upper boundary of the feasible region (Figure 4, lower surface and trajectory), while the COP_{heel} constraint determined the lower boundary (Figure 4, upper surface and trajectory), and the gravity constraint (Equation 1) determined the posterior boundary of the feasible region (Figure 3, upper right edge of the gray area).
Normal friction and strength constraints did not limit the feasible region unless friction levels were less than 0.82, dorsiflexion strength was reduced more than 51%, and plantar flexion strength was reduced more than 35%. Extreme alterations in friction and strength beyond these threshold values further reduced the feasible region (Figure 5).

The computer model used to derive these results was based on several simplifying assumptions, which therefore might limit its predictive capacity. For example, a single segment was assumed above the ankle joint. Movement termination was assumed to occur in the sagittal plane. It was assumed that to avoid an impending fall, a person may resort to instantaneous exertion of muscle strength at its maximum value (although for normal conditions, the model did not utilize maximum strength due to other constraints being more restrictive). The model also assumed that strength is dependent on joint position alone, and neglected the possibility that heel and toe rise could occur without initiating a fall. Overall, the effects of multisegmental interactions (i.e., the contributions of joints other than the ankle), of three-dimensional movement, of heel and toe rise, of joint velocity on strength limits, of rate-limits on muscle activation (cf. Zajac, 1989), and of other neurological constraints (cf., Nashner et. al., 1989) may influence the actual values of the predicted feasible region, and should be taken into consideration in future work. Two key concepts applied in this analysis -- the use of regional constraints to predict one's global ability to perform a functional task and the use of CM velocity-position interaction to understand dynamic balance -- are applicable to more complex and perhaps more insightful models than this endeavor.

Nevertheless, in spite of these limitations, the model predictions were consistent with previously published experimental data and trajectories derived from studies on gait, sit-to-stand (STS), STS followed by a volitional forward fall, and balance recovery from a horizontal bimanual pull. When the second foot touches the ground at the end of the final step in gait termination (Jian et al., 1993), the horizontal CM velocity-position combination was located well within the predicted feasible region (Figure 6, open square). Also as predicted, the ensemble average of the horizontal CM trajectories for both slow and fast STS (Pai and Rogers, 1990) started in the backward fall region E, but traveled out of it and into the feasible region D (Figure 6, solid lines) before subjects lost contact with the chair. In fact, the fast STS trajectory was located outside of the BOS when subjects lost contact with the chair (indicated by ⊗ in region D, Figure 6), yet was well within the feasible region. The velocity for the STS followed by a volitional forward fall (Pai and Lee, 1994) actually exceeded the feasible region before the instant of losing contact with the chair (indicated by circled X in region A, Figure 6). Finally, for the pendulum-like standing pull task (Michaels et. al., 1993), the horizontal CM velocity-position trajectory during balance recovery after releasing from the pull began outside of the BOS but well within the feasible region (short broken line, Figure 6).

Discussion

The analysis presented here demonstrates that an individual’s ability to terminate anterior movement can be estimated with a computer model when the anatomical, physiological, and environmental constraints are considered. The diagonal shape of the feasible region (Figure 3) indicates the interaction between the CM velocity and its position, i.e., a person can tolerate higher anterior velocities at more posterior CM positions without initiating a fall.

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Figure 6. Comparisons between the predicted velocity-position feasible region (a portion of the light gray diagonal band in Figure 3) and experimental data derived from: (1) gait (open square represents data taken from the time when the second foot touches the ground at the end of the final step in gait termination, Jian et al., 1993); (2) sit-to-stand in slow and fast speeds (trajectories with thin solid lines labeled as “Slow STS” and “Fast STS”, Pai and Rogers, 1990); (3) STS followed by a forward fall (trajectory with long broken line labeled as “STS+fall”; Pai and Lee, 1994); and (4) balance recovery after releasing from the pull (trajectory with short broken line, labeled as “Pull”, Michaels et al., 1993). Both forward fall region A (above shaded area, between toe and heel) and backward fall region E (below shaded area, behind heel), feasible region B (shaded area, between toe and heel) are located inside the base of support, while the forward fall region C (above shaded area, behind heel), feasible region D (shaded area, behind heel), and backward fall region E (below shaded area, behind heel), are behind the base of support. * indicates the instant of losing contact with the chair in STS.

The finding that normal strength limits did not play a restricting role was also consistent with previous work. Schultz and associates (1992) found that during STS, peak ankle torques were very similar in both young and elderly subjects (including those “unable” elderly who needed the assistance of their hands to rise from a chair), and were well below their reported maximum voluntary joint torques. Normal values of resultant ankle torques also did not play a restricting role in balance recovery mechanisms in model simulation of the standing human (Kuo and Zajac, 1993).

Furthermore, the findings of the present analysis are in qualitative agreement with recent observations on perturbed standing and subsequent stepping responses. Carhart and Yamaguchi observed that the faster and the further anterior the horizontal CM was moving after a perturbation, the more likely the person was to take a step (Carhart and Yamaguchi, 1995). This is consistent with the present analysis, which predicts that the feasible velocities become progressively reduced as the CM moves more anteriorly.

The final objective of a protective response after a perturbation (such as in stepping) is still to reduce the total body velocity, as in movement termination. Do and associates (1982) experimentally determined that the greater the perturbation, the greater the stepping velocity, and the greater the stepping length (a more anteriorly placed BOS). By stepping forward, a person effectively shifts the BOS in the anterior direction, presumably leading to an anterior shift in the upper boundary of the feasible region, and making it possible to terminate movement without taking any further steps.

Finally, the findings of this analysis may shed light on the understanding of the likelihood of taking a step when standing is perturbed (McIlroy and Maki, 1995). If the horizontal CM velocity-position combination is located in the fall regions (A, C or E, Figure 6), the model predicts that the person will have no option but to step or fall. On the other hand, the present model predicts that if the horizontal CM velocity-position trajectory is located inside the feasible region after a perturbation, the person still has the option of choosing a stepping response. This prediction makes no assertion about steps that might be initiated from the feasible region inside the BOS, as previous studies have speculated (McIlroy and Maki, 1993). Unfortunately, published reports have mostly concentrated on stepping frequency as a function of perturbation magnitude (McIlroy and Maki, 1993; Carhart and Yamaguchi, 1995), and no corresponding CM velocity-position trajectories have been found to verify the model predictions.

In summary, the traditional view that horizontal CM positions must reside inside the BOS to guarantee maintenance of balance in standing (Borelli, 1680; Dyson, 1977; Patla et al., 1990; Kuo, 1995) does not sufficiently define the feasible region for movement termination. This analysis has demonstrated that it is also critical to take into account the horizontal velocity of the CM, and has predicted the feasible region for successful movement termination to be a diagonal band in the horizontal CM velocity-position phase plane, located both inside and outside of the BOS. The basic conceptual framework demonstrated in this analysis might be applied to the study of segmental interactions in more complex movements, and might provide guidance for the evaluation of balance dysfunction, which, to date, has mostly been identified with the traditional view of balance control (O’Sullivan, 1994).

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APPENDIX: Model equations and constraints

A rigid body model of an inverted pendulum with a triangular base of support (the feet) was assumed (Figure 1). The equations of motion used were:

\[ \tau = m r^2 \dot{\theta} + m g r \cos \theta \]  
(A1)

\[ F_{gx} = m r (\dot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) \]  
(A2)

\[ F_{gy} = m r (\dot{\theta} \cos \theta + \dot{\theta}^2 \sin \theta) - m g \]  
(A3)

\[ COP = l f - \left[ \frac{b F_{gx} - \tau + cm g}{F_{gy}} + a \right] \]  
(A4)

\[ F_{gx} = F_{gy} \]  
(A5)

\[ F_{gy} = -m g = F_{gy} \]  
(A6)

where \( F_{gx} \) and \( F_{gy} \) were horizontal and vertical components of the ground reaction force; \( F_{gx} \) and \( F_{gy} \) were the resultant joint force components; \( m \), \( l f \), \( a \), \( b \), \( c \), and \( COP \) were the
foot mass, the length of the BOS, the distance between the ankle and the heel, ankle height, the distance between the CM of the foot and the ankle, and the distance between the COP and the toe; \( m, r, \Theta, \dot{\Theta}, \phi, \tau \) were mass, length, angular position, velocity, acceleration of the pendulum, and ankle torque.

Each of the BOS constraint inequalities (Equations 1 to 3) can be expressed as a boundary by taking its equation form and expressing it in terms of a relationship between the states (\( \Theta \) and \( \dot{\Theta} \)) and the torques. For example, the COP constraint (Equation 3) combined with (A4), substituting (A1) through (A3), resulted in:

\[
0 < b - \frac{\xi}{l} \sin \Theta + b \cos \Theta \sin \Theta - b m r \sin \Theta - m + m g \cos \Theta + a \leq b
\]

(A7)

The torques were then related to the states of the pendulum:

\[
\tau_{\text{opt, rev}} = [(b \sin \Theta + (l/2 - a) \cos \Theta) m g \cos \Theta + (l/2 - a) m g] / (l m g + m g)
\]

(A8)

\[
\tau_{\text{opt, heel}} = [(b \sin \Theta + a \cos \Theta) m g \cos \Theta + a (m + m g)] / (l m g + m g + a)
\]

(A9)

For the gravity constraint, Equation (1) was combined with Equations (A3) and (A6), and resulted in:

\[
\tau_{\text{gravity}} = m g r \cos \Theta + m r^2 \sin \Theta [\frac{(m + m g) \cos \Theta}{l m g + m g}]
\]

(A10)

Similarly, combining Equations (A1), (A3), (A5) and (A6) with Equation (2) gave the friction constraint:

\[
\tau_{\text{friction, rev}} = [(\sin \Theta - \mu \cos \Theta) m g r \cos \Theta + \mu r (m + m g)] / (\mu \cos \Theta - \sin \Theta)
\]

(A11)

\[
\tau_{\text{friction, pushover}} = [(\sin \Theta - \mu \cos \Theta) m g r \cos \Theta + \mu r (m + m g)] / (\mu \cos \Theta - \sin \Theta)
\]

(A12)

References


ERRATUM

Center of Mass Position-Velocity Predictions for balance control

Yi-Chung Pai and James Patton

In the above article published in the Journal of Biomechanics, 30(4):347-354, (1997), several typographical errors appeared in the equations. These errors only appeared in the final presentation of the equations, therefore did not affect the results. The correct equations are:

\[ \tau_{\text{friction \_ anterior}} = mgr \cos \Theta + \\
\left[ (\mu \sin \Theta + \cos \Theta)mr^2 \dot{\Theta}^2 - \mu rg (m_f + m) \right] / [\mu \cos \Theta - \sin \Theta] \quad (A11) \]

\[ \tau_{\text{friction \_ posterior}} = mgr \cos \Theta + \\
\left[ (\mu \sin \Theta - \cos \Theta)mr^2 \dot{\Theta}^2 - \mu rg (m_f + m) \right] / [\mu \cos \Theta + \sin \Theta] \quad (A12) \]

\[ \tau_{\text{gravity}} = mgr \cos \Theta + \\
mr^2 \dot{\Theta}^2 \tan \Theta - \\
\frac{(m_f + m)gr}{\cos \Theta} \quad (A10) \]

We would like to acknowledge Dr. K. Iqbal for bringing this to our attention.