Evaluation of a Model that Determines the Stability Limits of Dynamic Balance

James L. Patton
Department of Biomedical Engineering
Northwestern University
j-patton@nwu.edu
http://sulu.smpp.nwu.edu/~jim

Yi-Chung Pai
Programs in Physical Therapy
Northwestern University
c-pai@ nwu.edu

Wynne A. Lee *
Programs in Physical Therapy
Institute for Neuroscience
Northwestern University
wlee@casbah.acns.nwu.edu

This is an authors' preprint of the manuscript submitted to *Gait & Posture*, April 24, 1998, accepted September 17, 1998.

Abstract

A recent model of balance control has revealed two types of boundaries describing stability limits for center of mass (CM) dynamics: *torque boundaries* and *state boundaries*. The purpose of this study was to determine if these boundaries correctly characterize empirical data. We analyzed 2367 trials from 10 subjects who recovered their balance after they voluntarily pulled on a handle. We hypothesized that if model predictions were valid, both types of boundaries should encompass the empirical trajectories. We also hypothesized that each trajectory's nearest distance to the torque boundaries (the *torque safety margin*) would be correlated with the *center of pressure (COP) safety margin*, defined as the COP's nearest distance to the edge of the feet. The results supported the accuracy of the model-derived boundaries, with torque boundaries encompassing 100% and state boundaries encompassing 99.8% of the trials. Moreover, torque safety margins were highly correlated with COP safety margins, supporting the use of COP safety margins for estimating relative stability in dynamic tasks where balance is maintained. The distributions of the trajectories also suggested that a safety margin-oriented control strategy might be a robust alternative to the hypothesis that the central nervous system strives to optimize motion. The distinctions among different safety margins are discussed.

Keywords: Biomechanics, Dynamics, Movement constraints, Stance, Human, Posture, Voluntary movement, Balance control, Robust control, Optimal control.

Introduction

Little is known about how humans avoid falls during dynamic activities of daily life, although falling is a leading cause of death in the elderly [1]. This may be because no agreement has been made on how to quantify dynamic stability in postural control. One major theoretical problem is that there is a lack of a principled, mechanically rigorous basis for quantifying dynamic postural stability. This is because there many different ways to fall (slipping, stumbling, etc.), and because there are many different definitions for stability [2]. The purpose of this study was to evaluate whether a recently developed, mechanical model provides a basis for characterizing the kinematic and kinetic limits of stability.

Stability is commonly associated with the body's center of mass (CM) location relative to the base of support [3-8], although a large number of studies have also been conducted on various measures derived from the center of pressure (COP) [9-20]. In these studies, it is assumed that stability is associated with either 1) maintaining the CM over the base of support, 2) minimizing motions of the CM and COP, or 3) restoring the CM or COP to an optimal location. These assumptions are reasonably valid for quasi-static standing tasks, but dynamic modeling and experimentation have shown limitations. Stability can be maintained in some dynamic tasks where the CM and COP move considerably, and when the CM goes outside the base of support. For example, the CM to travels outside the base of support in locomotion [1, 21], in sit-to-stand [22, 23] and in pulls made while standing [24]. However, one simple fact is clear: leaning too far or too fast can result in a situation where it is not possible to recover balance. The crucial problem is to identify what constitutes unstable dynamics, and how such instability is related to the CM and COP motion.

Quantitative analysis with biomechanical models provide a rigorous approach for identifying which conditions are feasible for maintaining balance and which conditions lead to a fall [6, 25-28]. The inverted pendulum supported by a foot has often been used to characterize balance dynamics [1, 24, 29, 30]. Recently, a model of an inverted pendulum rotating about a triangular base of support (Figure 1, inset) was used to estimate the feasible combinations of the dynamic variables for CM control in the anterior-posterior direction (CM_{AP}) [25]. A fall, or instability, was operationally defined (as it is in the present paper) as anything that requires a change in the base of support, such as slipping, leaping, stumbling, or stepping. Pai and Patton (1997) defined two types of boundaries that describe the feasible limits of stability: *torque boundaries* and *state boundaries*.

The torque boundaries describe the range of torques beyond which the base of support (the model's "foot") begins to move (Figure 1; see also Figure 4 of Pai and Patton, 1997 [25]). For each instant in time, the ankle torque was evaluated relative to its maximum (dorsiflexion) and minimum (plantarflexion) allowable values. The allowable torques are dictated by several biomechanical constraints (avoiding slipping, keeping the COP within the base of support, avoiding leaping, and avoiding exceeding strength limits). Only the most restricting constraint dictates a torque boundary (Figure 1). The axes in Figure 1 span a larger range than is typical for static balance tasks that assume that the CM cannot be outside of the base of support. It can be considered a dynamic application of the "cone of stability" defined by McCullum and Leen [4]. Consequently, the model can characterize stability limitations for a broader range of dynamic tasks than is usually considered. If accurately modeled, the torque boundaries should encompass all empirical trajectories in which balance is preserved.

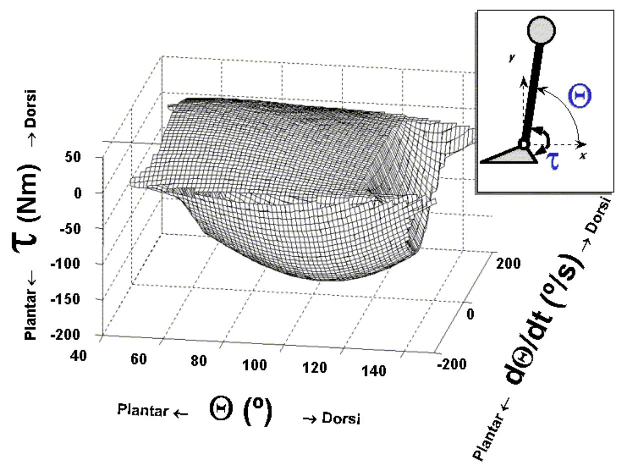

Figure 1: The torque boundaries for an inverted pendulum supported by a foot (see Appendix for derivation). For each position-velocity combination, a range of torques satisfies all biomechanical constraints. The resulting nonlinear surfaces (torque boundaries) describe the maximum plantarflexion and dorsiflexion torques, beyond which the feet begin to move. The approximate space these boundaries span is from -175 to 50 Nm in the torque dimension, from 46° to 135° in the position dimension, and ±175°/sec in the velocity dimension.

The other model-derived boundaries to stability were the state boundaries (shaded area in Figure 4 and Figure 3 in Pai and Patton, 1997 [25]). The state boundaries describe the range of position-velocity combinations beyond which it is impossible to recover balance (terminate CM motion to a point over the feet). When the CM moves beyond these states, the model predicts that a fall will occur at some time in the future. If accurately modeled, state boundaries should encompass all empirical trajectories in which balance is preserved. In summary, the torque boundaries define the conditions that cause the base of support to move, and the state boundaries define the conditions beyond which it is impossible to terminate movement during balance recovery.

These modeled boundaries, if valid, afford a principled way to evaluate relative stability during dynamic as well as static tasks. One way to assess this model's validity is to determine if the model-derived torque and state boundaries encompass all empirical trajectories when balance is preserved. Provided the torque boundaries represent the limits accurately, one measure of relative, dynamic stability could be obtained by calculating the torque safety margin, defined as how close the ankle torque comes to a boundary. Likewise, provided the state boundaries accurately represent actual limits, another measure of relative, dynamic stability could be obtained by calculating the state safety margin, defined as how close the CM states come to a boundary. However, the validity of these two kinds of model-derived boundaries has yet to be determined. The model assumes few mechanical constraints and it does not seem likely that a pendulum is capable of characterizing the effective dynamics of standing for dynamic, multijoint actions. Consequently, the approach may be too simple to fully describe stability limits.

* Correspondence author Address: 645 N. Michigan Ave, Suite 1100, Chicago IL 60611

Clearly, it is crucial to determine whether the boundaries adequately represent the limits to stable empirical data before using the proposed safety margins as measures of stability. While Pai and Patton (1997) showed that some representative, average trajectories lie inside the state boundaries, they did not evaluate a large number of individual trajectories.

If the boundaries do represent the limits of stability, a further question can be raised. Under normal biomechanical conditions, the model revealed that the size and shape of the torque boundaries were determined by the restriction that the COP must reside between the heel and the toe [25]. This suggested that relative stability could also be measured using the minimum distance between the COP and the boundaries of the feet (the _COP safety margin_), even for dynamic tasks. If the pendulum model effectively characterizes the limits to stability, then torque safety margins and the COP safety margins should be equivalent. Moreover, this would establish a link between COP safety margins and one mechanical cause of a falling. Several investigators have suggested the COP safety margin (also referred to by others as the stability margin) as a measure of static and dynamic stability [6, 7, 31-33]. To our knowledge, no one has demonstrated how well this measure is linked to the mechanical limitations of motions required to maintain balance.

The first purpose of this study was to investigate the validity of this modeling approach and the resulting model-derived boundaries of dynamic stability by testing the hypothesis that the torque boundaries and the state boundaries encompass empirical data. The second purpose of this study was to determine the equivalence of the torque and COP safety margins by testing the hypothesis that the two measures are significantly correlated. We used previously recorded data from subjects recovering their balance after pulling on a handle [24]. This pulling task provides a particularly challenging test of the model because it involves multijoint dynamics and visits extreme areas of dynamic space.

Methods

Protocol

The methods for this protocol have been described previously [24]. Briefly, ten healthy adults (23-49 years old, eight women and two men) with no history of orthopedic or neurological disorders volunteered to perform the pulling task (Figure 2) for five days. Before participating, each subject signed an informed consent form that conformed to federal and university guidelines. Subjects warmed up with light stretching to reduce the possibility of injury.

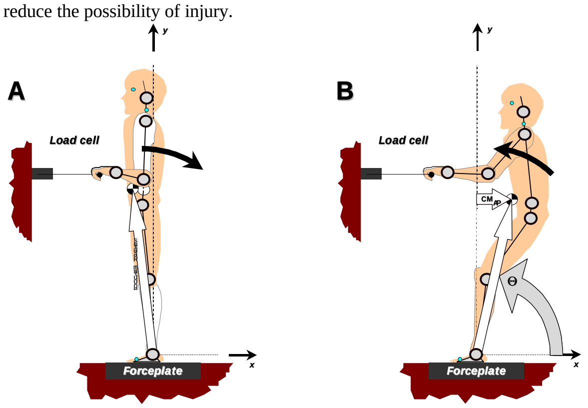

Figure 2: Pulling task and model variables. Small circles indicate locations of motion analysis markers and large circles indicate the location of joints used to determine the segmental motions. Pendulum radius (r) is shown in A and anterior CMAP position is indicated by the arrows (B). Subjects initiated motion from an upright position (A), developed posterior momentum (from A to B), generated a brief pull that reversed CM motion as they generated the pull and then recovered their balance (from B to A).

Subjects made a brief, horizontal pull on a handle to a range of target forces ranging from 20 to 80 percent of their maximum. They were instructed to keep their forearms parallel to the floor, their feet flat on the floor, and to begin and end in an upright posture. Before each pull, a cursor and target were displayed on a monitor to enable the subject to position their initial COP at a location 60% of the distance from the heel to the toe on the midline. Subjects could initiate movement at a self-determined time after an audible cue, but the pulling force they generated during the movement had to abruptly begin and end with zero force. Their goal was to try to generate a pull with a rise time of less than 150 milliseconds to a magnitude matching various peak force targets. Subjects were given verbal feedback on the duration and magnitude of the pull after completing the trial. A "fading" feedback schedule was used to inform the subjects about their force accuracy and timing (100% of the trials had feedback on Day 1, decreasing to 0% feedback by Day 5). In order to accomplish the pulling goal, subjects developed posterior momentum (Figure 2A), generated a brief pull that reversed CM motion (a bounce), and then recovered their balance (Figure 2B). Analysis focused on the balance recovery phase of the motion (Figure 3), after the subjects had ended their pull and were recovering to an upright stance. During this phase there was no longer any handle force acting on the body, which was consistent with the assumptions of the inverted pendulum model. Each subject attempted 243 trials, some of which were discarded due to technical problems. The highest number of trials discarded for any single subject was 41, and the least was zero. In all, 2367 trials from 10 subjects were recorded. Balance was fully recovered in all of these trials.

Instrumentation and data processing

Ground reaction forces and moments were recorded with an AMTI™ force plate at 200 Hz. COP was filtered with a third order, two-pass Butterworth filter with a cutoff frequency of 6 Hz. Pulling forces were recorded with an 1100N Sensotec™ load cell at 200 Hz. The onset balance recovery phase was determined by locating when the pulling force returned to the 95% confidence interval of baseline (zero) force recorded on the load cell.

An Elite™ video analysis system (BTS, Milan, Italy) recorded kinematics at 50 Hz during the first and last day of Experiment 1 and all of Experiment 2. Reflective hemispheric markers were located over the: fifth metatarsal, lateral malleolus, knee, head of the greater trochanter, superior iliac crest, neck, ear, temple, humeral head, lateral epicondyle of the elbow and the lateral bony prominence of the wrist (Figure 2, small circles). Marker trajectories were conditioned using a linear phase FIR filter [34]. Motion data were interpolated using a natural cubic spline algorithm to match the 200 Hz sampling frequency of the force plate and load cell data.

Empirical analysis

Ankle torque, _t_, was evaluated using force plate data and the center of the ankle joint using the following relation:

$$t = bF_{gx} - dF_{gy} \qquad \textbf{(1)}$$

where _b_ is ankle's vertical height, _d_ is the horizontal distance from the ankle joint to the center of pressure, and F_{gx} and F_{gy} are the horizontal and vertical forces measured by the force plate. This assumes the foot segment is stationary.

An eight-segment sagittal analysis determined the segmental motions of the foot, shank, thigh, pelvis, trunk, head, upper arm, and forearm based on an anthropometric regression model [35-37]. The vector sum of all segmental masses determined the CM trajectories, which were then differentiated using a two point, central differentiation algorithm. The resulting CM state trajectories were used for comparison with the model.

The _COP safety margin_ was defined as the nearest distance that the COP came to the boundary of the foot. Because the safety margin is the nearest distance to either boundary (the toe or heel), the range of safety margin values is 0.0 to 0.5, where 0.5 is the midpoint between the boundaries.

Model predictions

The methods for deriving the torque boundaries and state boundaries from a pendulum model were described previously [25, 38]. These methods are described briefly here and in more detail in the Appendix. The sagittal plane model consisted of two segments: a static base of support segment representing the feet and an inverted pendulum segment representing the motion of the effective dynamics of the CM. The angle Θ reflects the angle between the axis of rotation and the CM. The length of the pendulum was the height of the subject's CM when standing quietly. First, inequality restrictions were placed on the ground reaction forces, dictated by various mechanical constraints: avoid slipping, keep the COP within the base of support, avoid leaping, and avoid exceeding strength limits. Next, using the equations of motion, the inequality restrictions were related to the ankle kinetics, which were then related to the pendulum dynamics via the equations of motion. The result was the torque boundaries, which specify the range of ankle torques that do not cause the feet to move (Figure 1). Derivation of the state boundaries was also described previously [25]. Nonlinear optimization iteratively determined the fastest and slowest anterior velocities that still allow balance recovery (terminated motion to a point over the base of support). These extreme trajectories of balance recovery form the state boundaries. The resulting feasible region between the two boundaries is shown in gray in Figure 4.

Model evaluation using empirical measures

At each instant in time, a software program calculated the safety margins by comparing the empirical data to the model-derived boundaries. The torque safety margin for each trial was defined as the nearest distance between the empirical ankle torque and the torque boundaries. Likewise, the state safety margin for each trial was defined as the nearest distance between the empirical states and the state boundaries. These safety margins (like the COP safety margin) were expressed as a fraction of distance between the boundaries, resulting in a range from 0.0 to 0.5. Negative safety margins meant that the model-derived boundary was exceeded. Because balance was fully recovered in all recorded trials, a negative safety margin meant that the modeling failed to characterize the true limits of stability. Therefore, we tested the hypothesis that safety margins were more positive than negative.

The state safety margin measure potentially suffers from the mixing of units (m _vs._ m/s). This mixing can impose an arbitrary scaling effect, resulting in one set of units potentially yielding a smaller safety margin than the other. However, this problem is minimal because the two state boundaries are nearly parallel straight lines for the range of states studied (see Figure 4). Hence, the distance of a point to the boundary in one dimension (i.e., position) is linearly related to the distance of that point to the boundary in the other dimension (i.e., velocity). Therefore, measuring the state safety margin along either dimension and dividing it by the feasible range yielded equivalent results.

We also tested the hypothesis that the torque safety margin was related to the COP safety margin. Pearson correlation coefficients (r) determined their linear relationship. Significant and large r-values would imply that the torque safety margin could be accurately estimated by the COP safety margin.

Results

General observations

Subjects developed posterior momentum, made a brief pull that reversed CM_{AP} motion and then recovered balance (Figure 3). During the pre-pull and pulling phase of the motion, the CM_{AP} was often posterior to the base of support and outside of the torque- and state- boundaries. This was because the handle provided an alternative means of support not considered by the model. Had the cable suddenly broken, subjects would have fallen backwards. However, by the time the balance recovery phase began, empirical trajectories were within the torque- and state- boundaries. In the remaining sections we will consider how the empirical trajectories during the balance recovery phase relate to the model-derived boundaries.

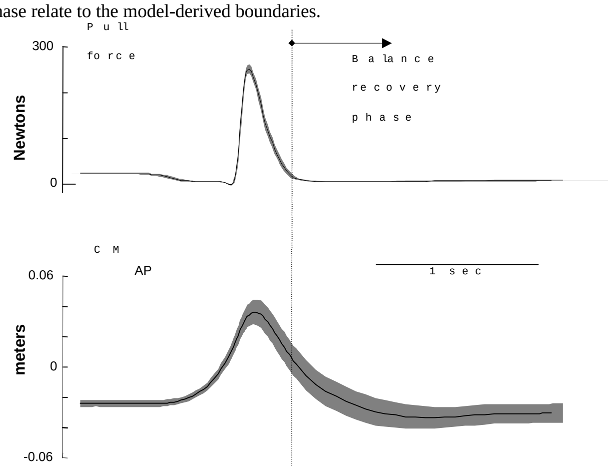

Figure 3: Ensemble- averaged pulling force and CMAP time records for one subject. Lines represent the ensemble-average, and shaded areas represent the 95% confidence interval.

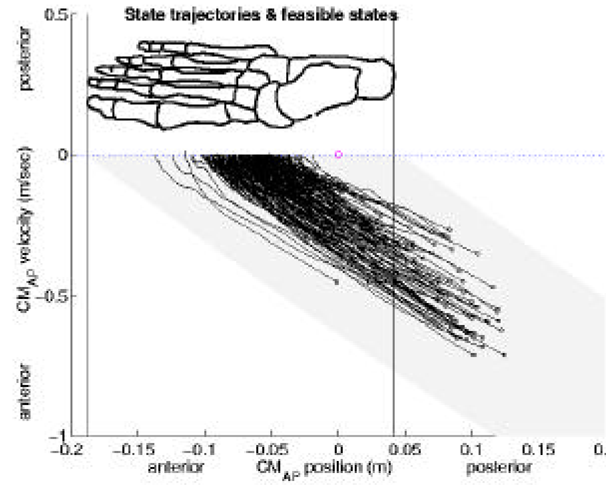

Figure 4: State trajectories of the CM for all 241 trials for one subject (different from Figure 3), superimposed on the state boundaries determined by the model (gray area), as described in the Appendix. Small circles indicate the beginning of the balance recovery phase.

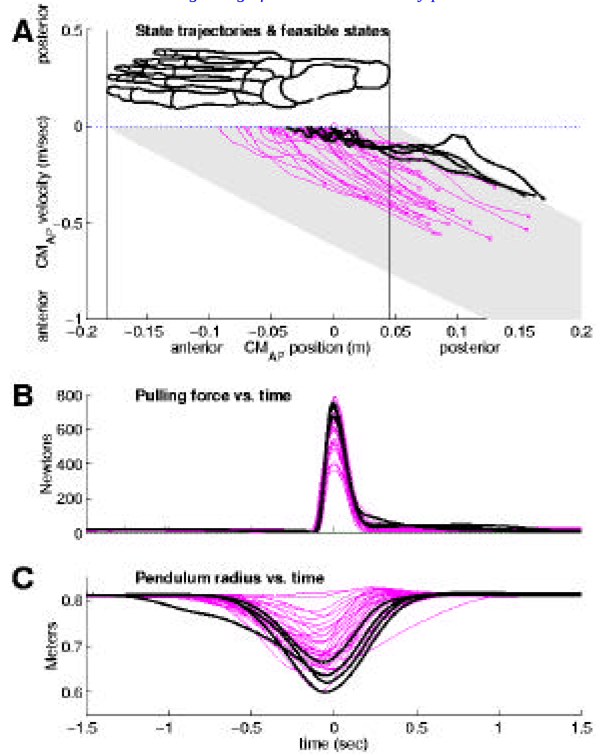

Figure 5: Figure highlighting the four trials from one subject (different from Figures 2 and 3) in which the model prediction failed (bold lines) along with all other trials (thin lines) at the same pulling force level for this subject. A) Phase plane trajectories during balance recovery. B) Pulling force time records. C) Pendulum radius ('suspensory') time records. Pendulum radius was defined as the distance between the ankle and the CM at each instant of time.

Do the model-derived torque and state boundaries encompass the empirical data?

The torque boundaries encompassed 100% of the empirical torque trajectories from the 2,367 trials (torque safety margins were all positive), supporting the torque boundaries as an effective description of the limits to stability. Trajectories were clustered near the center of the range between the boundaries. The median value of the torque safety margin was 0.41 of the range between the torque boundaries, with 95% of the values ranging between 0.21 to 0.49.

The state boundaries (edges of the gray area in Figure 4) encompassed 99.8% of the empirical state trajectories from the 2,367 trials (state safety margins were positive, except for five trials). Data from a typical subject is shown in Figure 4. As subjects recovered balance, their state trajectories generally traveled parallel to the state boundaries, maintained a roughly constant distance from the state boundaries and clustered near the center of the range between the boundaries. The median value of the state safety margin was 0.38 of the range between the state boundaries, with 95% of the values ranging between 0.22 to 0.48.

Five trials had trajectories that crossed the state boundaries (negative state safety margins), although balance was not lost. Hence, the model failed to predict the balanced movement in these trials. To further investigate why these failures occurred, we examined them more closely. Four of the five failures came from one subject (Figure 5A, bold lines). The state boundaries predicted that the subject should fall backwards when in fact she did not. We suspected that this subject might have 'cheated' by continuing to pull on the handle at low levels during the balance recovery phase, even though the pulling force should have been zero for this part of the motion. Analysis revealed that the subject pulled an average of 19 N during balance recovery in these failed trials (Figure 5B, bold lines). This was significantly larger than the average of 9 N, which the subject pulled during balance recovery in the other trials (Figure 5B, $p<0.05$). We also suspected that the subject had changed her effective pendulum radius -- a motion not represented by the model. The subject tended to squat during the motion (Figure 5C). Although the average drop in pendulum radius for the failed trials (6% drop; Figure 5C, bold lines) was larger than in this subject's other trials (5% drop; Figure 5C, thin lines) the difference was not significant. However, this subject significantly changed her radius (5% average) more than the rest of the subjects (2% average; $p<0.05$). Hence, some combination of pulling during balance recovery and squatting may cause the model to falsely predict a fall.

Relationship between the COP and torque safety margins

The results support the hypothesis that the modeled boundaries effectively characterized the limits to stability. This justified addressing the second question: whether COP safety margins, derived from the empirical data alone, are related to the torque safety margins, which are derived from a combination of modeling and empirical data. A strong relation between these two safety margins would support the COP safety margin as a valid measure of dynamic stability as well as further support the effectiveness of the pendulum model for characterizing the limits to stability.

Torque safety margins agreed well with COP safety margins (Figure 6). The median value for subjects' Pearson correlations between the two measures was 0.94 (ranging from 0.82 to 0.99). Hence, the two measures have a shared variance of about 88%. These high correlations imply that safety margins derived from the COP provide dynamic stability information that is comparable to the torque safety margins, which require a model to derive.

The method for determining the correlations between torque safety margins and COP safety margins could have led to erroneously high r-values. This is because both safety margins vary from zero to 0.5 and back to zero as a point moves from one boundary to another, which has the effect of taking an absolute value. Hence, it is mathematically possible for torque safety margins and COP safety margins to be perfectly negatively correlated, yet give an r-value of one. However, correlations between the normalized COP (0=toe; 1=heel) and normalized torque (0=lower torque boundary; 1=upper torque boundary) resulted in similar values to those based on safety margins (r-values averaged 0.93 and ranged from 0.85 to 0.98). Hence, we conclude that our methods did not falsely elevate the r-values, and that COP safety margins are valid measures of stability limits.

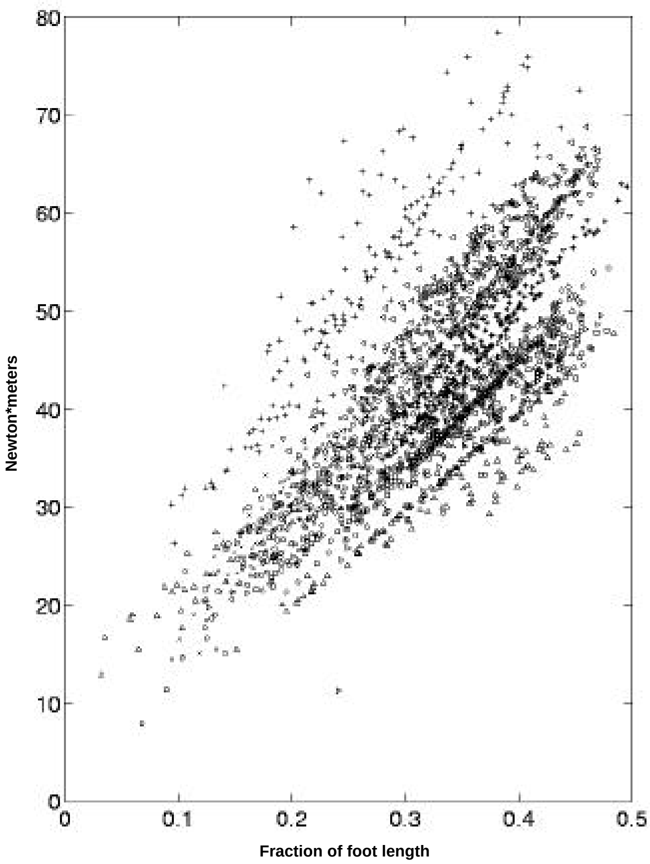

Figure 6: Correlation between the model-predicted, torque safety margins (vertical axis) and COP safety margins (horizontal axis). Each symbol represents a different subject, and each point represents a single trial. Note that the units of the torque safety margins and COP safety margins are different, so correlations do not have a slope of one. Because of difference in geometry and mass distributions, each subject's slope is different.

Discussion

The torque- and state- boundaries that were derived from the model showed excellent agreement with the experimental data, despite the high velocities and multijoint movements that occurred during the recovery of balance after pulling. These observations support the validity of the model for describing dynamic balance during this complex, multijoint action. In contrast, if the traditional assessment of relative stability had been used (CM must remain within the base of support [39]), the false conclusion often would have been made that subjects fell. This was because the CM_{AP} was temporarily *outside* of the base of support (Figure 4, trajectories to the right of the vertical lines). Thus, the model-derived torque and state boundaries more accurately assessed dynamic stability than boundaries based solely on CM location relative to the edges of the feet. The finding that the model correctly predicted dynamically stable behavior means that safety margins that are derived from torque and state boundaries offer a principled way to quantify dynamic stability. Moreover, the strong correlations between the COP and torque safety margins suggest that the COP safety margins offer one way to quantify dynamic stability, at least under some circumstances.

Limitations of the model and study

At least five assumptions may influence the accuracy of model predictions. First, the model assumes that the COP can reside anywhere between the heel and the toe, although approaching the heel or toe represents a physical impossibility. The COP actually represents a centroid of a distribution of pressures over an area, and if the feet are stationary, the COP cannot approach the heel or toe without reducing the area to a point. This may be one reason why subjects tend to step well before the COP reaches the heel or the toe [40, 41]. The COP typically resides within an area that is smaller than the full foot length (i.e., the 'functional' base of support) [42-44]. Hence, this assumption may have resulted in an overestimation of the feasible range between the boundaries. However, the length of the functional base of support varies with age and other unknown factors. Therefore, we used the unambiguous anatomical length of the foot to define the model's support base. A second model assumption was that the effective dynamics of this standing activity can be characterized by a model that only considers CM motion. A model that considers inter-segmental dynamics may be more accurate. A third model assumption was that CM motion, determined by cadaver-based regression estimates, was accurate. The body geometry and mass distribution estimates based on table data may have been better estimated via other methods [45-47]. The fourth model assumption was that manual (pulling) force was zero during balance recovery. Pulling force was non-zero during balance recovery in some trials, however, especially those trials where the model predicted a fall (Figure 5B). The fifth model assumption was that CM motion along the radius of the pendulum of is zero. This 'suspensory' motion was non-zero for some of the trials, and was more pronounced in those trials where the model predicted a fall (Figure 5C). For those trials where these last two assumptions were violated, the model and its predictions may not be applicable. Enhancing the model to include manual forces [24] and a suspensory degree of freedom [4, 8] should yield a more accurate prediction of stability for actions with significant manual forces or suspensory motions.

The present study is only an initial evaluation of the model's validity due to the limited scope of the experimental design. First, the study focused only on the case where subjects recovered balance. Second, we evaluated the model for only one task. It remains to be determined whether model validity generalizes to other tasks, such as rising from a chair, terminating gait, landing from a jump, or responding to an external perturbation. Other studies should seek to determine if the model also makes accurate predictions for other tasks and for falling behavior [22]. Large motions in multiple joints intuitively make a single inverted pendulum seem inappropriate, but the model predictions agreed nearly perfectly with the pulling task, which involves substantial motion at all those joints. This is consistent with other studies that have shown global models to characterize balanced movements quite well for a number of complex actions, including standing pulls, sit-to-stand, sway and locomotion [1, 24, 29, 30, 48-52].

While more work is needed to completely test the model's validity and generalizability, the model was clearly very successful overall in indicating that subjects recovered balance after making standing pulls. The remainder of the discussion considers the implications of the model for evaluating dynamic stability via torque, state and COP safety margins.

Evaluating stability using torque and state safety margins

The validity of the model supports using torque and state safety margins as a method for evaluating stability in dynamic actions. Safety (or stability) margins have been used for decades to measure static stability during robot locomotion. For example, McGhee and colleagues developed control algorithms for multi-legged gait that maximized the distance of the device's COP (equivalently, CM) from the edges of the feet after each step [31]. Under the more general treatment of the current dynamic model, the torque and state safety margins measure how close a person comes to the boundary between mechanically stable and unstable behaviors. Safety margins thus provide a measure of how dangerous a trajectory is, as opposed to how optimal. When torques and states are an acceptable distance from their boundaries (have large enough safety margins), balance may not be tightly regulated. Rather, control processes may occur only when the safety margins are small, a proposal consistent with several other recent motor-control studies [32, 51, 52].

Like the boundaries from which they are derived, the torque and state safety margins both provide different information about relative stability related to falling. Torque safety margins detect how close someone is to moving their feet (slipping, leaping or raising their heel or toe). State safety margins detect how close someone is to being incapable of recovering balance without shifting their base of support (e.g., stepping, grabbing onto something or pulling on the handle). Based on the approaches of this study, three logically distinct types of falls exist. One type of fall occurs when both safety margins are nonpositive: the feet move and balance cannot be recovered. A second type of fall occurs when just the torque safety margin is nonpositive. This type of fall, exemplified by the first step in gait initiation, occurs when the subject exerts such a large torque that the feet move, even though the state safety margin is positive. A third type of fall occurs when just the state safety margin is nonpositive. This situation predicts that a fall will occur sometime in the future, although the feet are currently stationary. An example of this type of predicted fall occurs when a person is suddenly perturbed, imparting too much momentum for CM motion to be successfully terminated over the base of support. The five discrepant trials might be other examples of this third type of fall, in which the CM may have moved outside the state boundaries. Under this interpretation, the additional pulling force (Figure 5B) represents the subject 'cheating' to recover balance that had been genuinely lost, rather than her failure to conform to the model's assumption of zero manual force.

In summary, either the torque or state safety margins can characterize relative stability when balance is successfully maintained (as in the present study), but both safety margins are needed to identify all types of falls. The next section discusses how the COP safety margin, a wholly empirical measure, can be used to quantify relative stability under some conditions.

COP safety margins

The finding that the COP and torque safety margins were highly correlated supported the hypothesis that the COP safety margins can provide a valid measure of relative stability for the recovery of balance after pulling. The COP safety margins are complementary to the more traditional COP measures such as peak-to-peak sway, mean sway path, sway path centroid or sway area [18, 19]. These traditional measures generally reference the COP to some presumed optimal state (e.g., the COP under midpoint of the foot, the COP under the axis of ankle rotation or the COP velocity at zero). The COP safety margin, in contrast, indicates how boundary for falling.

COP safety margins offer an advantage because they can be derived easily from force plate recordings. Torque and state safety margins require an estimate of the CM trajectory, typically requiring full-body kinematic recordings, which are often time-consuming. However, it should be noted that force plate recordings could also be used to obtain estimates of the CM trajectory as an alternative to using kinematic recordings [53-57]. An additional advantage to quantifying relative stability using COP safety margins is that no modeling assumptions are made. Torque and state safety margins require a complex computational model [25]. It remains to be seen if COP safety margins are accurate for movements that are poorly represented by a pendulum model.

Two cautions are warranted against relying solely on the COP safety margin to quantify relative stability. First, the validity of the COP safety margin rests on the assumption that the most critical constraint on stability is that the COP must reside between the heel and toe. This assumption is reasonable under conditions of "normal" friction, strength, or gravity. However, when friction is low ($\mu < .82$) and the shear forces are sufficiently high, the COP cannot approach the toe or heel without the foot slipping [25]. The COP safety margin would fail to predict such instability. Simulations also indicated that subjects such as the frail elderly with substantially reduced ankle strength (dorsiflexion torque reduced more than 51% of normal; plantarflexion torque reduced more than 35% of normal) may be unable to develop sufficient torques to drive the COP to extreme regions under the foot [25, 58]. The torque safety margin would still be valid in these cases because the model-derived boundaries can accommodate such altered mechanical conditions. The COP safety margin, in contrast, would not be valid because it does not change with altered friction, strength or gravity.

The second caution is that future falls cannot be predicted unless state safety margins are also considered. In some cases, the CM trajectory crosses a state boundary, indicating that the subject cannot recover balance without shifting the base of support. A fall is inevitable, yet the COP safety margin (i.e., the torque safety margin) could falsely indicate that the person is stable. It should also be noted that because the COP cannot exceed the boundary of the base of support, it cannot predict impending falls. In contrast, a state trajectory can exceed a state boundary, which is a clear indication of instability because a fall is forthcoming.

In summary, the present experimental findings, combined with previous modeling studies [25], provide empirical and theoretical grounds for using the COP safety margin as a valid measure of relative stability for tasks when biomechanical conditions are 'normal'. Other COP-derived measures, such as virtual-time-to-contact of the COP with its boundaries [59], may also yield important additional information about dynamic stability.

Implications of variability of torque and state trajectories in understanding balance control

Torque and state trajectories and their respective safety margins, although centered approximately on the midpoint of the base of support, were distributed widely within the boundaries that separate stable and falling regions of the torque-state space. This variability might reflect noise in the system. However, the variability also could indicate that the central nervous system organizes movements by selecting an operational subset of the feasible range of movements that are within acceptable safety margins for remaining stable [4, 60]. If the safety margin drops below a threshold, a corrective response could be triggered [61, 62]. With sufficiently large safety margins, no corrective response is needed. Such a control scheme would provide a robust strategy for maintaining stability, one that offers an alternative to the usual hypothesis that the central nervous system controls balance by organizing movements so that the CM (or COP) approaches some "optimal" trajectory [3, 63-66]. Future studies should investigate whether the central nervous system uses this robust strategy or optimal control to avoid falls.

Acknowledgements

This research was supported by the National Science Foundation (IBN-9319601), the Arthritis Foundation, and by the Whitaker Foundation. The authors thank Dhugal Bedford for his comments on an early revision of the manuscript. The authors also thank Fang Gao, Peter Pidcoe, and Aileen Russo for their technical assistance. We thank Suzanne Zolfo for her editing suggestions. Portions of this paper were presented at the 1997 IEEE Engineering in Medicine and Biology Society Meeting, Chicago.

Appendix:

This section briefly describes the derivation of the boundaries presented in Pai and Patton, 1997 [25], and reflects some typographical corrections [38]. The two-segment, sagittal plane, rigid body model of an inverted pendulum (representing the body CM) was connected by a pin joint to a triangular base of support (BOS, representing the feet) as shown in the inset of Figure 1. The model's dynamics represented the effective dynamics of the CM. The present study constructed separate, subject-specific geometric and mass distributions based on each subject's body measurements. Pendulum length was the height of the subject's CM while standing. BOS dimensions were obtained by direct measurement of the subject's feet. The foot mass of the feet was assumed to be 0.029*body mass (Dempster via [37]), and was subtracted away to obtain the mass of the pendulum segment.

Derivation of the *torque boundaries*

The derivation of the torque boundaries involved three steps. First, the constraints on the system were identified using mathematical inequalities. The BOS segment, through which the pendulum transmits forces to the floor, was assumed to have limited capabilities, imposing three "BOS constraints" on the mechanical behavior:

(1) Gravity Constraint: The BOS segment must not leave the floor (the net force on the foot must be positive):

$$F_{gy} \geq 0 \tag{2}$$

(2) Friction (Slip) Constraint: The shear force must not exceed the threshold:

$$\left|F_{gx}\right| < mF_{gy} \tag{3}$$

where μ is the coefficient of friction $\underline{m}$. (We assumed $\underline{m}$=1 for the friction between the feet and the floor for subjects in our laboratory [67].)

(3) Center of Pressure Constraint: The COP under the BOS must be maintained within the limits of its length:

$$0 < COP < l_f \tag{4}$$

where l_f is the length of the BOS (the "feet"). Thus, COP=zero at the anterior edge of the BOS (the "toe"), and COP=l_f at the posterior edge (the "heel").

The second step in the derivation of the torque boundaries was to combine the above equations with the following equations of motion:

$$t = mr^2\ddot{\Theta} + mgr\cos\Theta \tag{5}$$

$$F_{ax} = mr(\ddot{\Theta}\sin\Theta + \dot{\Theta}^2\cos\Theta) \tag{6}$$

$$F_{ay} = mr(-\ddot{\Theta}\cos\Theta + \dot{\Theta}^2\sin\Theta) - mg \tag{7}$$

$$COP = l_f - \left[\frac{bF_{gx} - t + cm_f g}{F_{gy}} + a\right] \tag{8}$$

$$F_{gx} = -F_{ax} \tag{9}$$

$$F_{gy} = m_f g - F_{ay}. \tag{10}$$

where $\underline{r}$ is pendulum length, $\underline{m}$ is pendulum mass, $\underline{g}$ is the acceleration due to gravity, $\underline{b}$ is the vertical distance from the floor to the joint, $\underline{c}$ is the horizontal distance from the BOS center of mass to the joint, $\underline{a}$ is the horizontal distance from the heel to the joint and $\underline{m_f}$ is the mass of the BOS. We rearranged and combined expressions into the following equations related to the inequality surfaces:

$$\begin{aligned} t_{cop\_toe} > \{[&b\sin\Theta + \\ &(l_f - a)\cos\Theta]mg\cos\Theta + \\ &[(l_f - a)\sin\Theta - \\ &b\cos\Theta]mr\dot{\Theta}^2 - \\ &(l_f - a)(m_f + m)g + cm_f g\}/ \\ &\{1 + \tfrac{(l_f - a)}{r}\cos\Theta + \tfrac{b}{r}\sin\Theta\} \end{aligned} \tag{11}$$

$$\begin{aligned} t_{cop\_heel} < \{[&b\sin\Theta - a\cos\Theta]mg\cos\Theta - \\ &[a\sin\Theta + b\cos\Theta]mr\dot{\Theta}^2 + \\ &a(m_f + m)g + cm_f g\}/ \\ &\{1 - \tfrac{a}{r}\cos\Theta + \tfrac{b}{r}\sin\Theta\} \end{aligned} \tag{12}$$

$$t_{gravity} > mgr\cos\Theta + mr^2\dot{\Theta}^2\tan\Theta - \frac{(m_f + m)gr}{\cos\Theta} \tag{13}$$

$$\begin{aligned} t_{friction\_anterior} > {}& mgr\cos\Theta + \\ &[(m\sin\Theta + \cos\Theta)mr^2\dot{\Theta}^2 - \\ &mrg(m_f + m)]/[m\cos\Theta - \sin\Theta] \end{aligned} \tag{14}$$

$$\begin{aligned} t^{friction\_posterior} < {}& mgr\cos\Theta + \\ &[(m\sin\Theta - \cos\Theta)mr^2\dot{\Theta}^2 - \\ &\ldots \end{aligned} \tag{15}$$

In addition to these constraints, we assumed that the limits of torque, $\underline{t}$, was related to the physiological limits on the ankle torque. Hence $\underline{t}$ was constrained to a range of values under maximum muscle strength based on a musculoskeletal model developed previously (Delp et al., 1990):

$$t^{maximum\_plantarflexion} < t < t^{maximum\_dorsiflexion} \tag{16}$$

The third step in the derivation of the torque boundaries is to render the mathematical intersection of inequalities 11-16. Only the constraints that influence the intersection define the torque boundaries, which are the inner, most bounding constraint surfaces (resulting in Figure 1).

Derivation of the *state boundaries*

The resulting torque boundaries were then used as a state feedback controller to determine the largest allowable braking torques at each instant in time. This system used nonlinear optimization in successive, forward dynamic simulations to iteratively converge on the *fastest* and the *slowest* state trajectories that still terminated at a position over the foot (i.e., to quiet standing). These two trajectories formed the state boundaries. Beyond the fast boundary, momentum is too great and a forward fall would be initiated. Likewise, beyond the slow boundary, momentum is too small and a backward fall would be initiated. The resulting feasible region between the two boundaries is shown in gray in Figure 4.

References

[1] Winter DA. *A.B.C.: Anatomy, biomechanics and control of balance during standing and walking.* , Waterloo, Ontario, Canada: Waterloo Biomechanics, 1995.

[2] Slotine J-JE and Li W. *Applied nonlinear control.* , Englewood Cliffs, NJ: Prentice Hall, 1991.

[3] Kuo AD. An optimal control model for analyzing human postural balance. *IEEE Transactions on Biomedical Engineering* 1995; 42: 87-101.

[4] McCullum G and Leen T. Form and exploration of mechanical stability limits in erect stance. *Journal of Motor Behavior* 1989; 21: 225-44.

[5] Mackinnon CD and Winter DA. Control of whole body balance in the frontal plane during human walking. *Journal of Biomechanics* 1993; 26: 633-44.

[6] Koozekanani SH, Stockwell CW, McGhee RB, and Firoozmand F. On the role of dynamic models of Quantitative Postureography. *IEEE Transactions on Biomedical Engineering* 1980; BME-27: 605-9.

[7] Murray MP, Seireg A, and Sholz RC. Center of gravity, center of pressure, and supportive forces during human activities. *Journal of Applied Physiology* 1967; 23: 831-8.

[8] Nashner LM and McCollum G. The organization of human postural movements: A formal basis and experimental hypothesis. *Behavioral and Brain Sciences* 1985; 8: 135-72.

[9] Patla AE, Frank JS, and Winter DA. Balance control in the elderly: implications for clinical assessment and rehabilitation. *Canadian Journal of Public Health. Revue Canadienne de Sante Publique* 1992; 83: S29-33.

[10] Goldie PA, Bach TM, and Evans OM. Force platform measures for evaluating postural control: reliability and validity. *Archives of Physical Medicine & Rehabilitation* 1989; 70: 510-7.

[11] Diener HC, Dichgans J, Bacher M, and Gompf B. Quantification of postural sway in normals and patients with cerebellar diseases. *Electroencephalography & Clinical Neurophysiology* 1984; 57: 134-42.

[12] Black FO, Wall Cd, Rockette HE, Jr., and Kitch R. Normal subject postural sway during the Romberg test. *American Journal of Otolaryngology* 1982; 3: 309-18.

[13] Geursen JB, Altena D, Massen CH, and Verduin M. A model of the standing man for the description of his dynamic behaviour. *Agressologie* 1976; 17: 63-9.

[14] Aggashyan RV, Gurfinkel VS, Mamasakhlisov GV, and Elner AM. Changes in spectral and correlation characteristics of human stabilograms at muscle afferentation disturbance. *Agressologie* 1973; 14: 5-9.

[15] Stevens DL and Tomlinson GE. Measurement of human postural sway. *Proceedings of the Royal Society of Medicine* 1971; 64: 653-5.

[16] Riley PO, Benda BJ, Gill-Body KM, and Krebs DE. Phase plane analysis of stability in quiet standing. *Journal of Rehabilitation Research and Development* 1995; 32: 227-35.

[17] Collins JJ and Luca CJD. Open-loop and closed-loop control of posture: A random-walk analysis of center-of-pressure trajectories. *Experimental Brain Research* 1993; 95: 308-18.

[18] Schieppati M and Nardone A. Free and supported stance in Parkinson's disease: The effect of posture and postural set on leg muscle responses to perturbation, and its relation to the severity of the disease. *Brain* 1991; 114: 1227-44.

[19] Kolleger H, Wober C, and Baumgartner C. Stabilizing and destabilizing effects of vision and foot position on body sway of healthy young subjects: A postureographic study. *European Neurology* 1989; 29: 241-5.

[20] Prieto TE, Myklebust JB, Hoffmann RG, Lovett EG, and Myklebust BM. Measures of postural steadiness: differences between healthy young and elderly adults. *IEEE Transactions on Biomedical Engineering* 1996; 43: 956-66.

[21] Jian Y, Winter DA, Ishac MG, and Gilchrist L. Trajectory of the body COG and COP during initiation and termination of gait. *Gait & Posture* 1993; 1: 9-22.

[22] Pai Y-C, Rogers MW, Patton JL, Cain TD, and Hanke TA. Static versus dynamic predictions of stepping following waist-pull perturbation in young and older adults. *Journal of Biomechanics* 1998; in press: .

[23] Pai Y-C, Naughton BJ, Chang RW, and Rogers MW. Control of body center of mass momentum during sit-to-stand among young and elderly adults. *Gait & Posture* 1994; 2: 109-16.

[24] Lee WA and Patton JL. Learning of coordination during multijoint pulls: Differences and similarities with a simple model. *Biological Cybernetics* 1997; 77: 197-206.

[25] Pai Y-C and Patton JL. Center of mass velocity-position predictions for balance control. *Journal of Biomechanics* 1997; 11: 341-9.

[26] Gordon ME. An analysis of the biomechanics and muscular synergies of human standing. Doctoral Thesis, Stanford University, 1991.

[27] Kuo AD. Visualization techniques for analyzing control of human movement: affine mappings between multi-dimensional spaces. Doctoral Thesis, Stanford University, 1992.

[28] Kuo AD and Zajac FE. Biomechanical analysis of muscle strength as a limiting factor in standing posture. *Journal of Biomechanics* 1993; 26S1: 137-50.

[29] Gurfinkel EV. Physical foundations of the stabilography. *Aggressologie* 1973; 14: 9-14.

[30] Hemami H and Golliday CL. The inverted pendulum and biped stability. *Mathematical Biosciences* 1977; 34: 95-110.

[31] McGhee RB and Frank AA. On the stability properties of quadruped creeping gaits. *Mathematical Biosciences* 1968; 3: 331-51.

[32] Hinton G. Parallel computations for controlling an arm. *Journal of Motor Behavior* 1995; 16: 171-94.

[33] Paloski WH and Nicholas SC, On estimating relative postural stability. Ninth Engineering Foundation Conference on Biomechanics and Neural Control, Deer Creek Ohio, 1996:65-6.

[34] D'Amico M and Ferrigno G. Technique for the evaluation of derivatives from noisy biomechanical displacement data using a model-based bandwidth-selection procedure. *Medical and Biological Engineering and Computing* 1990; 28: 407-15.

[35] Dempster WC, Gabel WJ, and Felts L. The anthropometry of manual work space for the seated subject. *American Journal of Physical Anthropometry* 1959; 17: 289-317.

[36] Plagenhoef S. *Patterns of human motion.* , Englewood Cliffs, NJ: Prentice-Hall, 1971.

[37] Winter DA. *Biomechanics and Motor Control of Human Movement.* 2nd ed: , 1990.

[38] Pai Y-C and Patton JL. Erratum: Center of mass velocity-position predictions for balance control. *Journal of Biomechanics* 1998; 31: 199.

[39] Borelli GA. *De Motu Animalium (Translated by Maquet, P., 1989).* , Berlin: Spinger Verlag, 1680.

[40] Maki BE and McIlroy WE. The role of limb movements in maintaining upright stance: the change-in-support strategy. *Physical Therapy* 1997; 77: 488-507.

[41] McIlroy WE and Maki BE. Task constraints on foot movement and the incidence of compensatory stepping following perturbation of upright stance. *Brain Research* 1993; 616: 30-8.

[42] Lee WA and Deming LR. Age related changes in the size of the effective support base during standing. *Physical Therapy* 1988; 68: 859.

[43] Blaszczyk JW, Lowe DL, and Hansen PD. Ranges of postural stability and their changes in the elderly. *Gait & Posture* 1994; 2: 11-7.

[44] King MB, Judge JO, and Wolfson L. Functional base of support decreaces with age. *Journal of Gerontology* 1994; 49: M258-M63.

[45] Sarfaty O and Ladin Z. A video-based system for the estimation of the inertial properties of body segments [see comments]. *Journal of Biomechanics* 1993; 26: 1011-6.

[46] Jensen RK. Estimation of the biomechanical properties of three body types using a photogrammetric method. *Journal of Biomechanics* 1978; 11: 349-58.

[47] Schaer AR, Baumann JU, and Meier G. Segmental Inertial Parameters - Determination and Effects on Normal and Pathological Human Locomotion. *First World Congress of Biomechanics, La Jolla, California, USA* 1990; 1: 96.

[48] Raibert MH. *Legged robots that balance.* , Cambridge, Mass: MIT Press, 1986.

[49] McMahon TA. Spring-like properties of muscles and reflexes in running. In: J. M. Winters and S. L.-Y. Woo eds. *Multiple Muscle Systems*. New York: Springer-Verlag, 1990: 578-90.

[50] Michaels CF, Lee WE, and Pai Y-C. The organization of multisegmental pulls made by standing humans: I Near-maximal pulls. *Journal of Motor Behavior* 1993; 25: 107-24.

[51] Chow CC and Collins JJ. Pinned polymer model of postural control. *Physical Review E* 1995; 52: 907-12.

[52] Jacobs R. Control model of human stance using fuzzy logic. *Biological Cybernetics* 1997; 77: 63-70.

[53] Levin OJM. An interative model for estimation of the trajectory of center of gravity from bilateral reactive force measurements in standing sway. *Gait & Posture* 1996; 4: 89-99.

[54] King DL and Zatsiorsky VM. Extracting gravity line displacement from stabilographic recordings. *Gait & Posture* 1997; 6: 27-38.

[55] Benda BJ, Riley PO, and Krebs DE. Biomechanical relationship between center of gravity and center of pressure during standing. *IEEE Transactions on Rehabilitation Engineering* 1994; 2: 3-10.

[56] Shimba T. An estimation of center of gravity from force platform data. *Journal of Biomechanics* 1984; 17: 53-60.

[57] Spaepen AJ, VranKen M, and Williams EJ. Comparison of the movements of the center of gravity and center of pressure in stabilographic studies. *Agressologie* 1977; 18: 109-13.

[58] Schultz AB. Mobility impairment in the Elderly: Challenges for Biomechanics Research. *Journal of Biomechanics* 1992; 25: 1383-91.

[59] Slobonov S, Sloubonova ES, and Newell KM. Virtual Time-to-Collision and human Postural Control. *Journal of Motor Behavior* 1997; 29: 263-81.

[60] Riccio GE. Information in movement variability about the qualitative dynamics of posture and orientation. In: C. Newell K.M., D. eds. *Variability and Motor Control*. Champagne, IL, USA: Human Kinetics Publishers., 1993: 317-57.

[61] Collins JJ. Random Walking During Quiet Standing. *Physical Review Letters* 1994; 73: 764-7.

[62] Hanneton S, Berthoz A, Droulez J, and Slotine JJE. Does the brain use sliding variables for the control of movements. *Biological Cybernetics* 1997; 77: 381-93.

[63] Uno Y, Kawato M, and Suzuki R. Formation and control of optimal trajectory in human multijoint arm movement. *Biological Cybernetics* 1989; 61: 89-101.

[64] Flash T and Hogan N. The coordination of arm movements: An experimentally confirmed mathematical model. *Journal of Neuroscience* 1985; 5: 1688-703.

[65] Nelson WL. Physical principles for economies of skilled movements. *Biological Cybernetics* 1983; 46: 135-47.

[66] Engelbrect SE and Fernandez JP. Invariant characteristics of horizontal-plane minimum-torque-change movements with one mechanical degree of freedom. *Biological Cybernetics* 1997; 76: 321-9.

[67] LLoyd DG and Stevenson MG. An investigation of floor surface profile characteristics that will reduce the incidence of slips and falls. *Mechanical Engineering* 1992; 17: 99-105.